Table 2 shows comparison to exact results by Heaslet and Warming.[2] Like the slab case, the trend is toward accuracy for small albedo and medium to small optical thicknesses. This is to be expected because of the source function equation, which has a slablike form in terms of $rS(r)$.

Table 3 illustrates the comparison of numerical results with those of Heaslet and Warming.[3] Exact results are unavailable for this problem ($\omega > 0$); however, the trend here is toward agreement for smaller albedo and some deviation as albedo approaches 1. Again, results for the dimensionless source function and exit intensity are presented by Kisomi and Sutton.[6]

All computations were performed on the Engineering Computer Center DEC VAX 11/780 at the University of Oklahoma. Computational times were very short in the slab and spherical cases; on the order of 1 s or less. The cylindrical case required numerical integration of the triple integral for the expansion coefficient and several other multiple integrations to compute the intensity and source function. This was done with two programs; one to compute the expansion coefficient and another to compute all parameters for a single optical thickness. In this fashion, the second computation was reasonably fast.

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Radiative View Factors from Spherical Segments to Planar Surfaces

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Introduction

THE determination of the configuration factor is a necessary step in computing the radiative interchange between two surfaces in the presence of thermally transparent media. In the calculation of radiative transfer from cryogenic tanks and spherical satellites to other surfaces such as fins and paddles containing solar cells, it is often desired to determine the view factor from spherical segments to planar surfaces.

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The view factor from spheres to nonintersecting planar surfaces was studied by Feingold and Gupta.[1] Later, Chung and Naraghi[2] obtained an exact solution for the view factor between spheres and intersecting coaxial annular rings. In some cases, the sphere is nonisothermal, and it was necessary to subdivide it into a number of isothermal spherical segments. A spherical segment is the part of a sphere bounded by two parallel planes that intersect the sphere or are tangent to it. The only available work in the literature for the view factor between spherical segment and planar surfaces was done by Ballance and Donovan.[3] They used the Monte Carlo method to obtain numerical values for the view factor between annular rings and hemispherical segments.

The current work develops a general formulation for the view factor between spherical segments and intersecting and nonintersecting planar surfaces. In its limiting case, the current approach can be used to evaluate the view factor from spheres and spherical caps to intersecting and nonintersecting planar surfaces.

Analysis

The analysis starts with the diffuse radiation from a differential area to a spherical segment. Then, making use of the reciprocity rule and rotation of the differential area about the centerline of the spherical segment, the view factor between the spherical sector and a coaxial differential ring is determined. The view factor between a spherical segment and a planar surface is then obtained by integrating the differential ring sector in a polar coordinate system.

View Factor Between a Differential Area and a Spherical Sector

Consider the configuration shown in Fig. 1. The center of the sphere is at the origin of the coordinate system. A spherical segment is generated by cutting the sphere by two parallel planes with equations $z = l$ and $z = h$ (surface A_2). Differential area $\mathrm{d}A_1$ is located at $x_1 = 0$, $y_1 = d$, and $z_1 = h$, and the unit vector normal to $\mathrm{d}A_1$ is parallel to the z axis, i.e., $\mathrm{d}A_1$ is located on the plane with the equation $z = h$.

Based on the contour integral method,[5] the view factor between $\mathrm{d}A_1$ and A_2 is given by

$$F_{\mathrm{d}A_1 - A_2} = l_1 \oint_C \frac{(z_2 - z_1)\,\mathrm{d}y_2 - (y_2 - y_1)\,\mathrm{d}z_2}{2\pi L^2} + m_1 \oint_C \frac{(x_2 - x_1)\,\mathrm{d}z_2 - (z_2 - z_1)\,\mathrm{d}x_2}{2\pi L^2} + n_1 \oint_C \frac{(y_2 - y_1)\,\mathrm{d}x_2 - (x_2 - x_1)\,\mathrm{d}y_2}{2\pi L^2} \qquad (1)$$

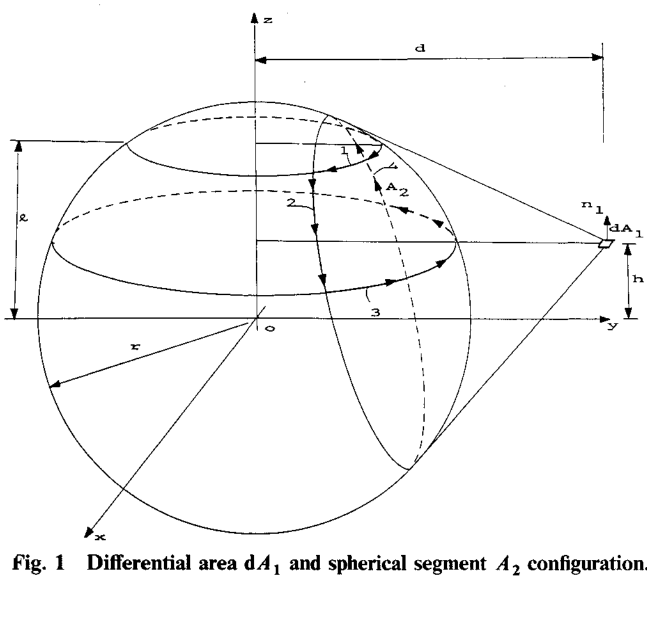

Fig. 1 Differential area $\mathrm{d}A_1$ and spherical segment A_2 configuration.

Table 1 Equations for the view factors from a differential point source to a spherical segment or cap

Region[2,3]	Condition	Equation
I	$D > \frac{1-LH}{\sqrt{1-L^2}}$ and $-1 < H < L$	$F_{dA_1-A_2} = \frac{1}{2\pi}\left\{\cos^{-1}\left(\frac{1-LH}{D\sqrt{1-L^2}}\right) + \frac{2(1-2L^2-D^2-H^2+2HL)}{\sqrt{(1+D^2+H^2-2HL)^2-4D^2(1-L^2)}} \times \tan^{-1}\left(\sqrt{\frac{(1+D^2+H^2-2HL+2D\sqrt{1-L^2})(D\sqrt{1-L^2}-1+LH)}{(1+D^2+H^2-2HL-2D\sqrt{1-L^2})(D\sqrt{1-L^2}+1-LH)}}\right) + 2\frac{\sqrt{D^2-1+2HL-(D^2+H^2)L^2}-\sqrt{D^2-1+2H^2-(D^2+H^2)H^2}}{D^2+H^2} + 2\frac{H}{(D^2+H^2)^{\frac{3}{2}}}\cdot\left[\sin^{-1}\left(\frac{H-(D^2+H^2)L}{\sqrt{H^2+(D^2-1)(D^2+H^2)}}\right) - \sin^{-1}\left(\frac{(1-D^2-H^2)H}{\sqrt{H^2+(D^2-1)(D^2+H^2)}}\right)\right] + 2\sin^{-1}\left(\frac{\sqrt{1-H^2}}{D}\right)\right\}$
II	$D < \frac{1-LH}{\sqrt{1-L^2}}$ and $-1 < H < L$	$F_{dA_1-A_2} = \frac{1}{\pi}\left\{\tan^{-1}\sqrt{\frac{1-H^2}{D^2+H^2-1}} - \frac{\sqrt{(D^2+H^2-1)(1-H^2)}}{D^2+H^2} - \frac{H}{(D^2+H^2)^{\frac{3}{2}}}\cos^{-1}\left(\frac{H\sqrt{D^2+H^2-1}}{D}\right)\right\}$
III	when $L > 0$ $D > \frac{1-LH}{\sqrt{1-L^2}}$ and $H < -1$ when $L < 0$ $D > \frac{1-LH}{\sqrt{1-L^2}}$ and $\frac{1}{L} < H < -1$ and $D > \frac{LH-1}{\sqrt{1-L^2}}$ and $H < \frac{1}{L}$	$F_{dA_1-A_2} = \frac{1}{2\pi}\left\{\cos^{-1}\left(\frac{1-LH}{D\sqrt{1-L^2}}\right) + \frac{2(1-2L^2-D^2-H^2+2HL)}{\sqrt{(1+D^2+H^2-2HL)^2-4D^2(1-L^2)}} \times \tan^{-1}\left(\sqrt{\frac{(1+D^2+H^2-2HL+2D\sqrt{1-L^2})(D\sqrt{1-L^2}-1+LH)}{(1+D^2+H^2-2HL-2D\sqrt{1-L^2})(D\sqrt{1-L^2}+1-LH)}}\right) + 2\frac{\sqrt{D^2-1+2HL-(D^2+H^2)L^2}}{D^2+H^2} + 2\frac{H}{(D^2+H^2)^{\frac{3}{2}}} \times \cos^{-1}\left(\frac{H-(D^2+H^2)L}{\sqrt{H^2+(D^2-1)(D^2+H^2)}}\right)\right\}$
IV	when $L > 0$ $D < \frac{1-LH}{\sqrt{1-L^2}}$ and $H < -1$ when $L < 0$ $D < \frac{1-LH}{\sqrt{1-L^2}}$ and $\frac{1}{L} < H < -1$	$F_{dA_1-A_2} = -\frac{H}{(D^2+H^2)^{\frac{3}{2}}}$
V	$D < \frac{LH-1}{\sqrt{1-L^2}}$ and $H < \frac{1}{L}$	$F_{dA_1-A_2} = \frac{1}{2\pi}\left\{1 + \frac{1-2L^2-D^2-H^2+2HL}{\sqrt{(1+D^2+H^2-2HL)^2-4D^2(1-L^2)}}\right\}$

where $l_1 = m_1 = 0$, $n_1 = 1$, and the contour changes depending on the position of the differential area relative to the spherical sector. In general, the contour consists of four lines as shown in Fig. 1. Contour lines 1 and 3 are the intersection with the sphere of the planes with equations $z = l$ and $z = h$, respectively, and contour lines 2 and 4 are the intersection with the sphere of tangent lines through dA_1. At certain positions of differential area dA_1 one or two of the contour lines vanish. Five regions of solutions can be distinguished as shown in Figs. 2 and 3. These regions of solutions are generated by plane $z = l$, tangent to the sphere at $z = l$ and the horizontal plane tangent to the sphere at $z = -r$.

For region I, the plane of differential area dA_1 intersects the spherical segment, and Eq. (1) has to be evaluated along the four contour lines shown in Fig. 1. When the differential area is in region II, the spherical segment appears as a complete sphere from dA_1 and line 1 vanishes from the contour shown in Fig. 1.

When $H < -1$ $(H = h/r)$, the plane of the differential area does not intersect the sphere, and the line 3 vanishes from the contour shown in Fig. 1 (region III in Figs. 2 and 3). For region IV, a full sphere is visible from dA_1.

When $L < -1$ $(L = l/r)$, there is an extra solution for the view factor from the differential area to the spherical segment. This solution corresponds to the region V in Fig. 3. From region V, the spherical segment appears as a disk, and the contour is only line 1.

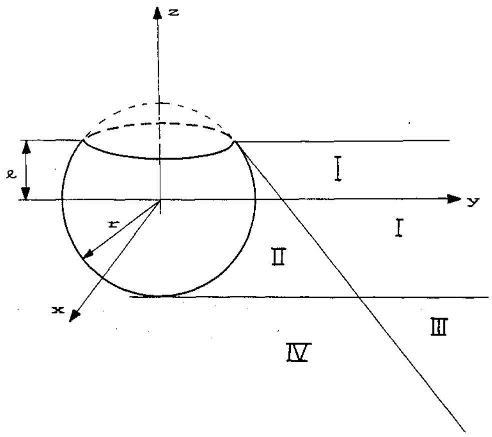

Fig. 2 Regions of validity of equations for the view factor from differential area to spherical segments when ($l > 0$).

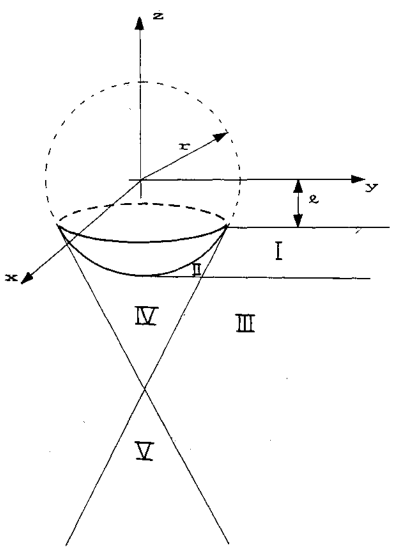

Fig. 3 Regions of validity of equations for view factors from differential areas to spherical segments when ($l < 0$).

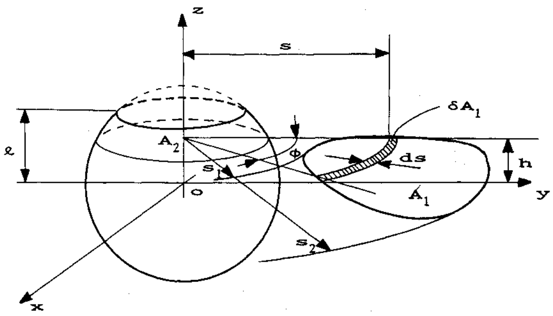

Fig. 4 Differential ring sector—planar surface and spherical segment configuration.

Table 1 shows the resulting view factors for the five aforementioned regions in terms of dimensionless variables $D = d/r$, $L = l/r$ and $H = h/r$. In the solutions, it is assumed that the plane of differential area dA_1 is parallel to the xy plane, i.e., the normal to dA_1 is parallel to the z axis.

View Factors from Spherical Segments to Planar Surfaces

Consider the configuration shown in Fig. 4. A differential ring sector is generated by rotating the differential area dA_1 about the z axis. The rotation angle is ϕ, and the view factor from the spherical segment to the differential ring sector is given by

$$dF_{A_2-\delta A_1} = \int_0^{\phi} \frac{sds}{2\pi r(l+r)} F d_{A_1-A_2} d\phi$$

$$= \frac{\phi sds}{2\pi r(l+r)} F_{dA_1-A_2} \tag{2}$$

where $F_{dA_1-A_2}$ is given in Table 1. Notice that the plane of the differential ring sector is parallel to the xy plane.

Next, it is assumed that the differential ring sector is a part of a planar surface in a polar system of coordinate. Then, the view factor from the spherical sector to surface A_1 is given by

$$F_{A_2-A_1} = \int_{s_1}^{s_2} dF_{A_2-\delta A_1}$$

$$= \frac{1}{2\pi r(l+r)} \int_{s_1}^{s_2} \phi(s)\, sdsF_{dA_1-A_2} \tag{3}$$

where $\phi(s)$ is the functional representation of the contour of surface A_1 in the polar system of coordinates. Equation (3) can be used to evaluate view factors from spherical segments to planar surfaces as long as the contour equation of the corresponding planar surfaces in the polar or Cartesian system of coordinates is available. It should be noted that the formulation given by Eq. (3) in its limiting case when $l = r$ can be used to evaluate the view factor from spheres or spherical caps to the intersecting and nonintersecting planar surfaces.

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